SURVEY DESIGN USING INDIVIDUAL NUMERICAL SCALES IN THE FRAMEWORK OF ANALYTIC HIERARCHY PROCESSES

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Abstract. *This paper discusses the adequacy of a generalization of Saaty's 1-9 scale proposed by Liang at all (2008) in the attempt to identify individual scales. Several surveys in completely different areas were conducted on different topics. Comparisons among the consistency index-as a measure of a "good answer" and the previously mentioned scale reveal a non monotonic correspondence among those two criterions. Also, the individual scale considered – which is in itself a generalization of other similar scales for measuring individual responses – is not uniquely determined for a single respondent and is very often contradictory. Yet, the potential benefits in determining individual scales of measurement are enormous and maybe the most important one is getting rid of the myth of the good appliance of the "law of large numbers" in social sciences.*

Keywords: Analytic Hierarchy Processes, knowledge sharing, mapping, numerical scale, simulated annealing, verbal responses.

1. Introduction

The determinacy of a convenient way of measuring individual responses – checked on a large range of surveys-can help in getting rid of the questions of the type: is the sample size relevant? or "how the selection of the participants at this survey was done?" in favor of the remarks of the next type: "the respondents are highly inconsistent, therefore we cannot trust the results obtained", or "the experts asked provided consistent answers and they conjecture that….". It is long recognized that the surveys on different topics conducted in the spirit of analytic hierarchy process (AHP) (Saaty, 1977) and asking questions in comparative terms and in terms of intensity are better suited when individuals with utility functions not checking the transitivity and invariance axioms are participating. Also, in the literature of behavioral economics, starting with Kahneman and Tversky (1981,1984) there is a consistent stream showing that utility's transitivity and invariance assumed among the axioms introduced by Newman and Morgenstern are often not fulfilled and dependent of the context. *Dominance* in between two alternatives A and B is established through the question „Which alternative you prefer out of A and B?". Preference's *intensity* is established through the question:" On a scale from…to … by how much you prefer the alternative you chose to the other one? ". The original scale proposed by Saaty is a 1-9 numeric one, as shown in Finan and Hurley (1999).Even if there is a one to one correspondence between this scale and any other finite one, alternative sets of scales were proposed in time, in the attempt to alleviate the relationship between the numeric scale and linguistic choices. Also, alternative numerical scales were discussed from the point of view of obtaining acceptable sets of priority values. An open research issue is the way in which the choice of a numerical scale can impact any AHP analysis. This paper addresses the problem of individual's scales of measurement, this is, given a fixed numerical scale, say for example the original 1-9 one-it is an open question whether the phrase "moderately preferred" refers to value 3 (as is in the original scale) for all the individuals. According to Liang et all (2008), suppose that a decision maker (DM) states that he/she moderately preferred alternative A to B and moderately prefers alternative B to C. We then ask the DM to what extent he/she prefers alternative A to C. With this information one can identify his/her individual scale $S_t(k)$ by selecting a suitable value of t. The scale analyzed in this paper is $S_t(k)=(9+tk)/(10+t-k)$, with $t>-1$. As $t\to\infty$, $S_t(k)\to k$ which means that individuals with high values of the parameter t are closer to the uniform, original scale of Saaty's.

If k_{AB} is the numerical judgment corresponding to the verbal judgment between alternatives A and B, in this reference paper the authors distinguish between consistency of preference judgments between alternatives A, B and C, expressed through the possibility to determine the individual scale out of the equation statement $S_t(k_{AB})\ S_t(k_{BC}) = S_t(k_{AC})$ and the possibility to allow a degree of inconsistency of preference judgments through the usage of a specified parameter ε which can be determined out of the inequalities $(1-\varepsilon)S_t(k_{AC})<S_t(k_{AB})S_t(k_{BC})<(1+\varepsilon)\,S_t(k_{AC})$.

In this paper the focus will be on the analysis of the determination of the parameter t out of the equation statement $S_t(k_{AB})\,S_t(k_{BC})=S_t(k_{AC})$ for a single application of the AHP in a problem, in order to illustrate the problems raised by this scale.

2. Individual scales in a particular experiment

The experiment used for the illustration of some problems raised in determining a good measure of individual scale is based on the development of a simple hierarchy model with one goal and five criterions. Using the terminology in AHP, the goal is to determine a vector of priorities among few factors determining the demand of life insurances at the ING Company, Bucharest. The criterions are as follows: *legislation* (C1), *trust in ING* (C2), *general trends in state pensions-pillar 1*(C3), *perception of risk* (C4) and *revenue* (C5). The questions asked make the distinction between dominance and intensity. The dominance is established through a question of the type:" In your decision to buy a life insurance at the ING Company which one is more important: *legislation* or *trust in ING*? "The intensity is established by the next question: "On a scale from 1 to 9 (1-equally important, 9-extremely important) by how much is more important in your decision the criterion you mentioned in comparison to the other one?" The survey was delivered among the ING's clients who bought a life insurance. Every respondent had to answer 10 questions setting the dominance among the five criterions and 10 questions setting the intensity of their preferences. The survey was designed and conducted electronically, using a professional account in the Survey Monkey template.

Previously described paired comparison judgments in the one-layer hierarchy are summarized in a matrix of judgments. Matrix of judgments is determined assuming values equal to one on the main diagonal and also reversibility of the preferences-so that if C_1 is preferred to C_2 at a corresponding absolute value of 5, the C_2 will be preferred to C_1 at an absolute value of 1/5, which is 0.2. The corresponding vector of priorities is calculated in an eigenvalue formulation. The solution is obtained by raising the matrix to a sufficiently large power, then summing over the rows and normalizing to obtain the priority vector. The process is stopped when the difference between components of the priority vector obtained at the k-th power and at the (k+1) power is less than some predetermined small value. The vector of priorities is the derived scale associated with the matrix of comparisons (Saaty, 1977). Proportionality among the answers is measured in the Saaty's original framework through a consistency index (CI) which is desirable to be less than 0.1. In the situation of high values of this CI, Satty recommends that the DM is asked again the same questions. Starting from the matrix of preferences built on the basis of the firs respondent's answers, I will illustrate how the parameter t in the individual scale of measurement

$S_t(k)$ can be determined, the problems raised and the relation with the associated CI. The CI was calculated using the SuperDecisions Software.

Suppose that for a decision maker, shortly referred as Decision Maker 1(DM1) the next matrix of choices is built, as presented in Table 1:

Table 1

Matrix of pair wise comparisons for decision maker 1-DM1

	C1	C2	C3	C4	C5
C1	1	0.16666667	0.2	0.2	0.125
C2	6	1	5	0.14285714	0.16666667
C3	5		1	5	0.16666667
C4	5	7		1	0.2
C5	8	6	6	5	1

As it can be noticed, apart of the unitary elements on the main diagonal and the values belonging to the range {1,2… 9}, the rest are to be completed according to the reciprocity condition, so that if the a_{31} element in the DM1 matrix is 5, the a_{13} element is going to be 1/5.

If one is considering the A choice being the C5 criterion (A:C5), B choice being the C4 criterion (B:C4) and the C choice being the C1 criterion (C:C1) then $k_{AB} = 5$, $k_{BC} = 5$ and $k_{AC} = 8$. The compulsory condition (CC) indicated in the Liang et all (2008) paper, namely $k_{AB}\ k_{BC} > k_{AC}$ (CC) is obviously satisfied and as well it is satisfied the supplementary condition (SC) indicated in the Appendix of this paper, namely $k_{AB} \leq k_{BC} < k_{AC}$.

(SC) The individual parameter t is determined from the condition $S_t(k_{AB})$

$S_t(k_{BC}) = S_t(k_{AC})$ (eq. 2.1)

with

$S_t(k) = S_t(k) = (9+tk)/(10+t-k)$ (eq. 2.2)

and the value obtained is $t_{DM1} = 1.1693$. This value is greater than minus 1, so the compulsory condition for the determination of an individual scale is fulfilled.

What one could do, admitting that she believes that this is the secret determination of the DM1?

The next step would be to map all the DM's answers into the $S_{1.1693}(k)$ map, rewrite accordingly the matrix of pairwise comparisons and derive the correspondent vector of priorities.

In Table 2 is presented the matrix of the DM1's mapped answers ($DM1_{individual\ scale}$) into the $S_{1.1693}(k)$ map:

Table 2

The matrix of the DM1's mapped answers (DM1$_{individual\ scale}$) into the $S_{1.1693}(k)$ map

	C1	C2	C3	C4	C5
C1	1	0.32276252	0.41553902	0.41553902	0.17267249
C2	3.0982531	1	2.4065129	0.24261133	0.32276252
C3	2.4065129	0.41553902	1	2.4065129	0.32276252
C4	2.4065129	4.1218190	0.41553902	1	0.41553902
C5	5.7913104	3.0982531	3.0982531	2.4065129	1

The vectors of priorities corresponding to the DM1 and DM1$_{individual\ scale}$ matrices are presented in Table 3.

Table 3

Vectors of priorities corresponding to the DM1 and DM1$_{individual\ scale}$ matrices

Criterions	Vector of priorities for DM1 matrix	Vector of priorities for DM1-individual scale matrix
C1 (legislation)	0.047882735	0.064140310
C2 (trust in ING)	0.13291354	0.15444047
C3 (general trends in state pensions)	0.14305907	0.17420389
C4 (perception of risk)	0.15321899	0.19814399
C5 (revenue)	0.52292566	0.40907133

Continuing in the same line of reasoning, instead of aggregating vectors of priorities coming from brute matrices of pairwise comparisons, it could be aggregated vectors of priorities for the correspondent individual scale matrices. Therefore, there is no longer needed a large number of respondents, since each of them was adjusted corresponding to its individual scale. Apart of this, respondents can be grouped corresponding to the individual levels of consistency, instead of exterior, predetermined criterions like age, sex, education, so on so forth. So, the huge advantage of determining a good individual scale through which any individual subjectivity can be better assessed in the framework of a survey becomes now clearer.

In the following will be examined the shortcomings of the particular method presented in Liang et al. (2008) paper, thus opening few directions for further improvements.

3. The indeterminacy of the parameter reflecting individual choices for decision matrices with higher dimension than 3

One can notice that in an example as the one presented above, if there are more than three criterions (or alternatives) in an hierarchy, the decision matrix corresponding to the aggregation of these criterions has a dimension strictly greater than 3 and therefore there are at most four different ways of considering the A, B, C choices and consequently at most four different values for the parameter t, reflecting individual choices.

In the example presented above, apart of the determination done initially corresponding to a value of the individual parameter t equal to1.1693, one could consider the other possible next alternatives: A = C5, B = C2, C = C1 with $k_{AB} = 6, k_{BC} = 6, k_{AC} = 8$. This choice is also checking both compulsory (CC) and supplementary (SC) conditions, namely $k_{AB}\, k_{BC} > k_{AC}$ and $k_{AB} \le k_{BC} < k_{AC}$. The corresponding value of the parameter t is –0.25, negative but still in the range $(-1,\infty)$. Table 3 is completed with the vector of priorities corresponding to the DM1-individual scale for t = –0.25 as shown in Table 4:

Table 4

Vectors of priorities to the DM1 and $DM1_{individual\ scale}$ matrices for t=1.1693 and t=-0.25

Criterions	Vector of priorities for DM1 matrix	Vector of priorities for $DM1_{individual\ scale}$ matrix, t=1.1693	Vector of priorities for $DM1_{individual\ scale}$ matrix, t=-0.25
C1 (legislation)	0.047882735	0.064140310	0.095530538
C2 (trust in ING)	0.13291354	0.15444047	0.16968256
C3 (general trends in state pensions)	0.14305907	0.17420389	0.18714776
C4 (perception of risk)	0.15321899	0.19814399	0.21321601
C5 (revenue)	0.52292566	0.40907133	0.33442314

As it can be noticed, there is still no problem about rank reversal, yet for the choice of A = C2, B = C3, C = C1 correspond ing to $k_{AB} = 5, k_{BC} = 5, k_{AC} = 6$, also checking $k_{AB}\, k_{BC} > k_{AC}$ and $k_{AB} \le k_{BC} < k_{AC}$, we get the value t = –1.272, which is no longer in the range $(-1,\infty)$. The consistency index for this DM1 matrix is $CI_1 = 0.6540$.

For the second respondent in this survey (DM2 matrix), there is only one combination of criterions, A = C2, B = C3, C = C1 with $k_{AB} = 4, k_{BC} = 4, k_{AC} = 7$ checking both the compulsory and the supplementary conditions for which the corresponding t is t = 2.1623. The consistency index for DM2 matrix is $CI_2 = 0.5573$. Also, there are 5 more combinations of criterions checking only the compulsory condition k_{AB} k_{BC}> k_{AC} for which the corresponding t-values are out of the admissible range.

For the third respondent (DM3 matrix) there are two combinations of criterions A,B and C checking both the compulsory and supplementary conditions and yielding to the same value of the parameter t, t = –0.6 and three more other combinations checking only the compulsory condition with values for the parameter t out of the admissible range. The consistency index for DM3 matrix is $CI_3 = 0.4703$.

For the forth respondent there is no combination of criterions A, B and C checking both compulsory and supplementary conditions and $CI_4 = 1.1020$ and for the fifth respondent there is only one combination of criterions A, B and C checking compulsory and supplementary conditions with t = 1.246 and two other combinations of criterions checking the compulsory condition yielding values of the parameter t outside the admissible range. The consistency index for DM5 matrix is $CI_4 = 0.4715$.

Therefore, is obvious that the way it is introduced in the original paper Liang et all (2008), the determination of the individual scale of measurement is not uniquely determined for a certain decision matrix of a dimension strictly higher than three.

4. Distortions in the relation consistency index-individual Parameter t in the determination of an individual numerical scale

The determination of the priority vectors corresponding to positive matrix-a decision matrix-relay on the general finding that proper values $(w_i)_{I=1,2..n}$- corresponding to positive, reciprocal matrices $A = (a_{ij})_{i,j=1,2...n}$ checking $a_{ij}a_{jk} = a_{ik}$ and $a_{ij} = a_{ji}$ –have the next property:

for every i,j =1,2,..n, every element a_{ij} in the A matrix is the ratio w_i/w_j, where $Aw=wI_n$.

Therefore, the matrix A with $a_{ij}a_{jk}=a_{ik}$ is regarded as being a decision matrix in which every element represents the ratio among the importance associated with criterion i (w_i) to the importance associated with criterion j (w_j) .

If a certain reciprocal decision matrix A is not consistent, yet the difference $a_{ij}a_{jk}-a_{ik}$ is acceptable low, for every i,j,k, then it can be shown that correspondent changes in the proper vector are still holding the difference $a_{ij}-w_i/w_j$ sensible low. Another important result from the matrix theory of interest in the further considerations is that if $\lambda_1, \lambda_2, \ldots \lambda_n$ are the proper values corresponding to the proper vectors $w_1, w_2, ..w_n$ so that $\lambda_{max} = {}_{max}\{\lambda\ i\}_{i=1,..n}$ then the matrix A is consistent if and only if $\lambda_{max} = n$ and always holds true the inequality $\lambda_{max} \geq n$. As a result, a measure for the departures from the consistency condition is the consistency index CI introduces as being the ratio between (λ_{max} -n) and (n-1): $CI = (\lambda_{max}-n)/(n-1)$ .High values of the consistency index indicate high changes in understanding and therefore instable decisions. An acceptable amount of inconsistency, quantified by the condition $CI < 0.1$ is explained by the fact that new knowledge also requires slightly changes in the preferences. According to Saaty, "*Assuming that all knowledge should be consistent contradicts experience that requires continued revision of understanding*".

The consistency condition for a certain decision matrix is therefore alleviated trough the determination of an individual numerical scale expressed through the individual parameter t >–1 so that $S_t(k_{AB})\ S_t(k_{BC}) = S_t(k_{AC})$. Given also the fact that as $t \to \infty$, $S_t(k) \to k$ which means that individuals with high values of the parameter t are closer to the uniform, original scale of Saaty's, one would expect that for a consistent decision maker, its decision matrix expressed in the original Saaty's scale would correspond to a high value of the parameter t and a low value of the consistency index.

So, it should be observed an inverse, monotonic relationship among the individual parameter t and the corresponding consistency index.

In the previous section there were observed the following relations among the individual parameter t and corresponding consistency index:

$t = 2.1623$, $CI_2 = 0.5573$

$t = 1.246$ $\quad CI_4 = 0.4715$

$t = -0.6$, $\quad CI_3 = 0.4703$

Apart of the previous findings, one can also check the following:

If $a_{12} = 2, a_{23} = 5, a_{13} = 5$ then CI = 0.051 and t = –0.989 ,

if $a_{12} = 2, a_{23} = 7, a_{13} = 6$ then CI = 0.077211 and t = –0.989,

if a12 = 2,a23 = 3,a13 = 2 then CI = 0.1288 and t = –0.989.

This shows that apart of the fact that the inverse monotonic relationship among consistency index and parameter t is not working, also the determination of the individual parameter t in the context of the analyzed individual numerical scale cannot distinguish among quite different preferences.

Regarding the problem of the determination of an unitary individual numerical scale among a hierarchy, in section 2 it was illustrated the situation in which for a decision matrix of a dimension greater than 3 there is no unique determination for the individual parameter t. The problem can be extended to a whole hierarchy and to the answers provided in the context of an survey by a single respondent. It is necessary therefore to be able to come up with a single representation of the individuals' answers, for the whole hierarchy considered.

Thus, one can determine the uniquely determined t^* for the whole particular hierarchy considered so that the sum of the distances between the highest proper value and the correspondent matrix dimension is at minimum.

The distance between the initial vector of priorities of a certain respondent and the vector of priorities recalculated using the numerical scale optimally determined for the particular respondent under consideration will enter as a measure of spread around the mean of optimized vector of priorities and the sum of all individual's spread will be a measure of the standard deviation around the mean.

5. Conclusions and further research directions

This paper highlights the potential benefits of determining good individual numerical scales in the framework of AHP and in line with the findings of Liang et al. (2008). On the other hand it shows on a particular example corresponding to a one layer hierarchy with five criterions that major identification problems can occur in the attempt to apply the theoretical results in the previously cited paper. Also, there is a irregular dependence between the consistency measured through the consistency index and the individual consistency measure from the previously discussed scale. As a conclusion, the problem of calculating individual scale for more than three alternatives should be in attention in the subsequent developments of this idea. Also, there is an urgent need to a more unitary definition and calculation of the consistency's choices for a certain decision maker.

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